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ON ELECTRONIC SPACE CHARGE WITH HOMOGENEOUS INITIAL ELECTRON VELOCITY BETWEEN PLANE ELECTRODES

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Zusammenfassung.

In Erweiterung der bisherigen Arbeiten über diesen Gegenstand haben wir die im Gitter-Anodenraum infolge Raumladung (virtueller Kathode) umkehrenden Elektronen nicht vernachlässigt, nachdem sie diesen Raum verlassen haben. Ein Teil dieser Umkehrelektronen gelangt ein zweites, drittes, ... Mal in den Gitter-Anodenraum und beeinflusst den Potential- und Stromverlauf dort stark. Die möglichen Kurvenformen sind in einer Anzahl von Fällen aufgezählt worden, wobei in den betrachteten Fällen Vollständigkeit erstrebt wurde.

I. *Introduction.* We consider an arrangement, consisting of a plane cathode, emitting electrons of zero initial velocity, a grid, built of parallel rods or of a maze network and a plane anode. The grid will be such, that practically no scattering of the electrons, passing through its mazes, occurs. This means a rather fine grating. The grid has the positive tension V_1 against the cathode and the anode the positive tension V_2. The grid corresponds to the so called screen grid in a radio tube. The distance grid-anode is d. The space grid-anode will be called V-space for sake of briefness. We are concerned with the electronic space charge phenomena in the V-space. Secondary electron emission of the grid or of the anode is not taken into account. Several discussions on this space charge have been published [1] [2] [3] [4] [5] [6]. A number of characteristics of this space charge are discussed here, that have not been published before.

The chief extension, given here to existing theory, is connected with returning electrons. In the previous publications, electrons which — after passing through the grid on their way to the anode — are returned to the grid by the space charge barrier in the V-space, are left out of discussion after they have got out of the V-space. In reality a portion of these returning electrons will be absorbed by the grid. Those, which pass through the grid make their way into the cathode-grid space and the electrons which are not absorbed by the cathode space charge cloud are a second time attracted by the positive potential of the grid plane and, if they are not captured by the grid wires, may pass a third time through the grid and enter the V-space a second time. Some electrons may repeat this motion several times. These electrons, revolving around the grid wires, cause a considerable alteration of the space charge in the V-space. This is expressed in the characteristic curves for the current densities and the potential distribution connected with the V-space.

II. *Fundamental equations.* Let the current density (Amp cm^{-2}) entering the V-space be i_0 and the current density on the anode be i_2. We assume, that all electrons, composing the current density i_0, have exactly equal velocities on entering the V-space, this velocity being given by the tension V_1 of the grid. Furthermore it is assumed, that i_0 is not influenced by returning electrons, except in the way, expressed by eq. (4).

The three possible potential distributions in the V-space are: 1°. No potential minimum occurs in the V-space. All electrons, entering this space, pass on to the anode. 2°. A potential minimum occurs in the V-space, but the potential V_m of this minimum is positive. Again all electrons, entering the V-space, pass on to the anode. 3°. A potential minimum of zero potential occurs in the V-space. Some electrons, entering the V-space, are returned at this potential barrier.

Considering case 3, if V denotes the potential in the V-space and x a coordinate, perpendicular to the planes of the cathode, the grid and the anode, we have $dV/dx = 0$ at this minimum and also $V = 0$. Hence the electron velocity is zero and the current density is finite, which corresponds with an infinite space charge density. Physically this state of affairs is hence impossible. It has to be considered as a mathematical fiction. Its bearing on real circumstances must be sought in the fact, that the electrons have a certain velocity distribution in stead of a single velocity. The idealization corresponding with this single velocity may lead to an approximate picture of current- and potential distribution in the V-space, as long as certain details, one of which was mentioned above, are disregarded. A more correct but also more complicated treatment of this problem, taking into account the electron velocity distribution, will be published at a later date.

As the first potential distribution, mentioned above, may be considered as trivial, we shall assume, that a potential minimum exists in the V-space and we shall count x from this minimum, the positive direction being towards the anode. The distance of this minimum from the anode is called x_2 and the distance from the grid is called x_1.

The differential equations of the problem are:

$$\left.\begin{aligned} \frac{d^2V}{dx^2} &= -9.10^{11}\,4\pi\rho; \\ v^2 &= 2\frac{e}{m}V; \\ \rho &= -i/v, \end{aligned}\right\} \tag{1}$$

where ρ is the space charge density in Coulomb cm^{-3}, V the potential in Volts, x in cm, $e/m = 17{,}6\,.\,10^{14}$ cm^2 sec^{-2} Volt^{-1} the ratio of specific charge and mass of an electron, v the electron velocity in cm/sec, i the current density in Amp cm^{-2}. As we have 5 variables, V, x, ρ, v, i, we may eliminate 2 variables, e.g. ρ and v. The boundary conditions are: $V = V_1$ at $x = -x_1$ and $V = V_2$ at $x = x_2$, whereas $dV/dx = 0$ at $x = 0$. The current density i is everywhere in the V-space equal to i_0, if $V_m > 0$. If $V_m = 0$, i is equal to i_2 between the potential minimum and the anode. The space between the potential minimum and the grid is in this case discussed below.

In the case that $V = V_m > 0$ at $x = 0$, the solution is:

$$V = \left\{Bi_0\left(\frac{3x}{Ai_0} + \sqrt{\frac{9x^2}{A^2i_0^2} + \frac{8v_0^3}{A^3i_0^3}}\right)^{2/3} + \right.$$
$$\left. + Bi_0\left(-\frac{3x}{Ai_0} + \sqrt{\frac{9x^2}{A^2i_0^2} + \frac{8v_0^3}{A^3i_0^3}}\right)^{2/3} - V_m^{1/2}\right\}^2 \tag{2}$$

where

$$mv_0^2 = 2e\,V_m,\; A = 9.10^{11}\,4\pi\frac{e}{m} \text{ and } B = \frac{A}{2}\left(\frac{m}{2e}\right)^{1/2}$$

Equating V to V_1 and to V_2 at $x = -x_1$ and $x = x_2$ respectively, V_m may be calculated as a function of V_2/V_1 and of i_0.

If $V_m = 0$ the current densities on both sides of the potential minimum will generally be different. Of a number of electrons, arriving at $x = 0$, coming from the grid, a part α will pass through the potential minimum, whereas a part $(1-\alpha)$ will return to the grid.

In the region between the potential minimum and the anode eqs. (1) yield the well known eq. of Langmuir:

$$V = \frac{m}{e} \frac{A^{2/3} i_2^{2/3}}{8} (6x)^{4/3}, \tag{3}$$

where i_2 denotes the current density between the minimum and the anode. If i_0 is the current density, composed of electrons, which enter the V-space for the first time, a part $(1-\alpha)i_0$ is returned at the potential minimum. Of this part a quantity $\beta(1-\alpha)i_0$ will arrive a second time at the potential minimum, coming from the grid. This process is repeated, giving rise to a total current density from the grid to the potential minimum of:

$$i_p = i_0 \{1 + \beta(1-\alpha) + \beta^2(1-\alpha)^2 + \ldots.\} = \frac{i_0}{1-\beta(1-\alpha)} \tag{4}$$

The current density from the potential minimum to the anode is

$$i_2 = \alpha i_0 \{1+\beta(1-\alpha)+\beta^2(1-\alpha)^2+\ldots.\} = \frac{\alpha i_0}{1-\beta(1-\alpha)} = \alpha i_p. \tag{5}$$

The current density returning from the potential minimum to the grid is:

$$i_1 = i_0(1-\alpha)\{1+\beta(1-\alpha)+\beta^2(1-\alpha)^2+\ldots.\} =$$

$$= \frac{i_0(1-\alpha)}{1-\beta(1-\alpha)} = (1-\alpha)\, i_p = i_p - i_2. \tag{6}$$

The eqs. (1) may be solved for the region between the grid and the potential minimum in the same way as for the region between the minimum and the anode, if we take $\rho = -(i_p + i_1)/v$ in the former case. The solution is:

$$V = \frac{m}{e} \frac{A^{2/3}(i_p + i_1)^{2/3}}{8} (6x)^{4/3}. \tag{7}$$

Equations 2—7, upon proper discussion, yield all the characteristic curves of the problem.

III. *Some characteristic curves including the case* $\beta = 0$ *(no revolving electrons) and the case* $\beta = \frac{1}{2}$. In view of the extensive collection of characteristic curves, contained in the existing literature, only those curves will be discussed here, which satisfy one or both of the two conditions: 1° curves, directly suited for experimental verification, 2° curves, the character of which has, in our opinion, not been pointed out sufficiently in previous articles. The calculations, necessary for obtaining each of the curves, are direct derivations from the equations in section II and, being of an elementary character, are omitted here.

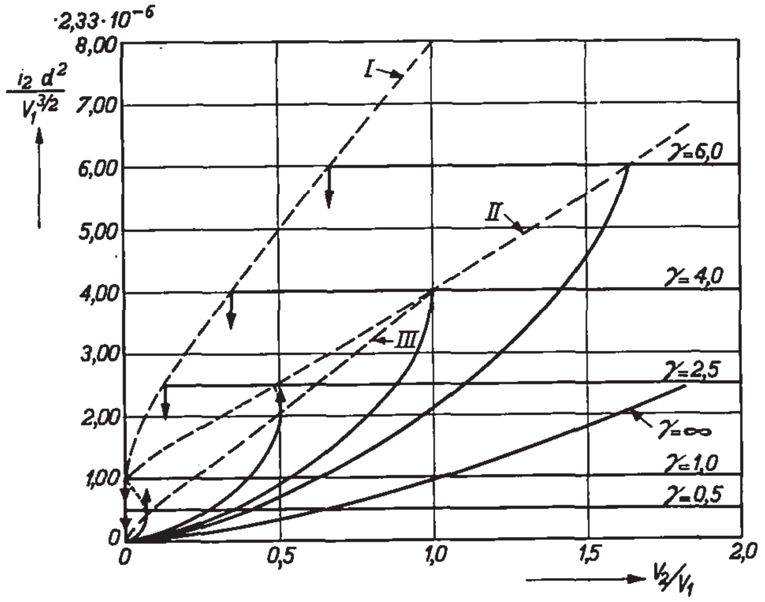

Fig. 1. Vertical scale: $i_2 d^2/V_1^{3/2}$, where i_2 is the anode current density (Amp/cm^2), d the distance grid-anode (cm) and V_1 the grid tension against the cathode (Volts). Horizontal scale: V_2/V_1, where V_2 is the anode tension against the cathode (Volts). Full curves for different values of $\gamma = i_0 d^2/2{,}33\ 10^{-6}\ V_1^{3/2}$, where i_0 is the electron current density (Amp/cm^2) composed of electrons, which enter the grid-anode space for the first time. Dotted curves marked I, II and III are boundaries of different regions for the full curves. In this fig. $\beta = 0$.

In fig. 1, which holds for $\beta = 0$, the current density i_2 is shown as a function of V_2/V_1 for different values of

$$\gamma = \frac{i_0\, d^2}{V_1^{3/2} \,.\, 2{,}33 \,.\, 10^{-6}}$$

We consider V_1 as constant and let V_2 increase from zero onwards. In the case $\gamma = \infty$ the current density i_2 increases steadily. Hence, there exists a potential minimum of zero potential in the V-space. Such a potential minimum is sometimes called a virtual cathode. In the case $\gamma = 6$, the current density i_2 increases with increasing V_2 until $i_2 = i_0$, which occurs about at $V_2/V_1 = 1{,}63$. After this point a further increase of V_2 leaves $i_2 = i_0$ unaltered, which means, that all electrons, entering the V-space, pass on to the anode. No virtual cathode exists in this region. Now, if V_2 is decreased, there are two possible solutions of the equations between V_2/V_1 about 1,63 and V_2/V_1 about 0,66. One solution is $i_2 = i_0$, while the other coincides with the i_2-curve considered for increasing values of V_2. Hence, in this double valued region there may or may not exist a virtual cathode. Transition from one state of affairs to the other may occur at any point of this region. The boundary curve I fixes the lowest value of V_2/V_1, for which the equation $i_2 = i_0$ may be satisfied, for any given value of γ. The boundary curve II marks the highest value of V_2/V_1, at which a virtual cathode may exist, resulting in a current density i_2, which is smaller than i_0. At the boundary curve II we have $i_2=i_0$ and $V_m=0$, which equations determine this curve. Below $\gamma = 4$, the curve II loses its practical value, as the i_2-curve has a vertical tangent $\partial i_2/\partial V_2 = \infty$ at a value of i_2 smaller than i_0. A new boundary curve III must be substituted for II, if $0 \leq \gamma \leq 4$. This curve III is determined by the equation $\partial i_2/\partial V_2 = \infty$. Fig. 1*a* is similar to fig. 1 but holds for $\beta = \frac{1}{2}$. The value $\beta = \frac{1}{2}$ was measured by the authors for several commercial valves. We refer to a future publication for these measurements.

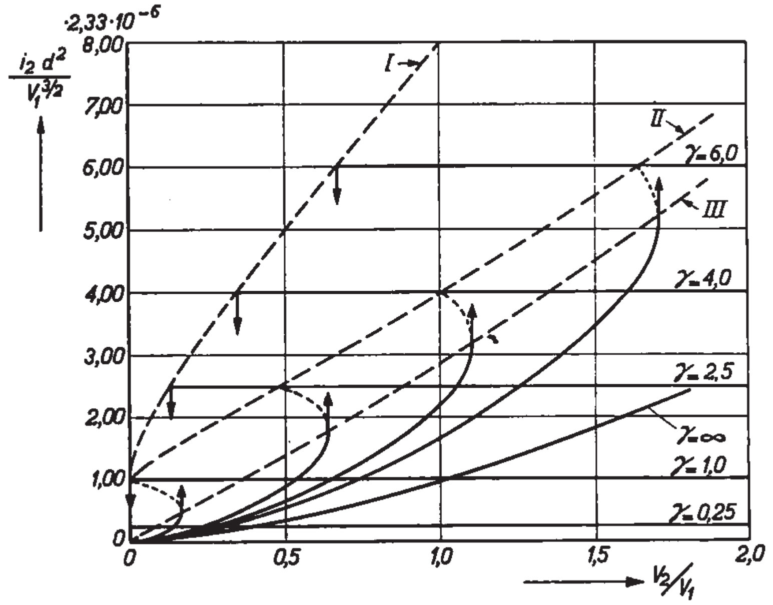

Fig. 1*a*. Same as fig. 1, but for $\beta = 1/2$, whereas fig. 1 holds for $\beta = 0$. Here β is the portion of the electrons, returning before the space charge barrier in the grid-anode space, which arrive a second time before this barrier.

In fig. 2 the potential V_m at the potential minimum is shown as a function of V_2/V_1 for $\gamma = 6$. A virtual cathode exists, if V_2 starts from zero onwards until V_2/V_1 is about equal to 1,63 for $\beta = 0$. Then abruptly, V_m/V_1 rises to a finite value and continues to increase, if V_2/V_1 is further increased. In the region between V_2/V_1 about 0,66 and 1,63 two states can exist, one with and one without virtual cathode. Transition from one state of affairs to the other may occur for any value of V_2/V_1 in this region. The boundary curve I marks the point V_2/V_1 about 0,66, where the V_m-curve has a vertical tangent. This vertical tangent necessitates an abrupt transition of V_m if V_2 is decreased below this value.

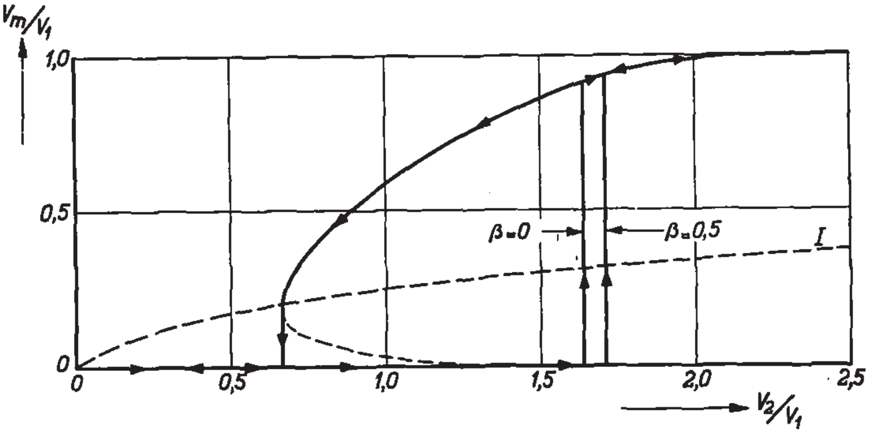

Fig. 2. V_m/V_1 as a function of V_2/V_1, where V_m is the potential of the potential minimum in the grid-anode space (Volts). The quantity $\gamma = 6$ in this figure.

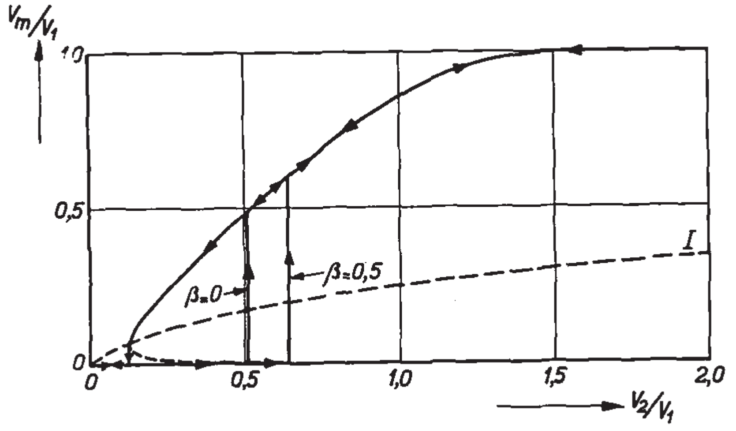

Fig. 3. As fig. 2 but for $\gamma = 2,5$.

Fig. 3 is essentially equal to fig. 2, but is calculated for $\gamma = 2,5$, whereas fig. 4 holds for $\gamma = 1,0$.

In fig. 5 V_m/V_1 is shown as a function of γ (which is $i_0 d^2/V_1^{3/2} \, . \, 2{,}33 \, . \, 10^{-6}$) at $V_2/V_1 = 2{,}0$. If γ is increased from zero onwards, V_m starts at the value $V_m = V_1$ and then decreases steadily until $V_m/V_1 = [\sqrt{V_2/V_1} \, / \, (1 + \sqrt{V_2/V_1})]^2$ and at this point a virtual cathode, with $V_m = 0$, is abruptly formed. The boundary curve I marks the point, where the V_m-curve has a vertical tangent.

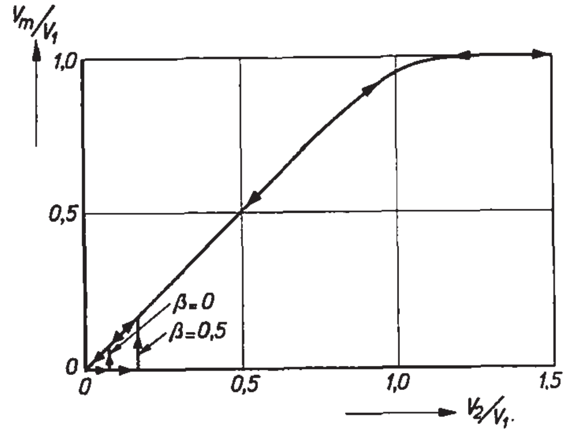

Fig. 4. As fig. 2 but for $\gamma = 1$.

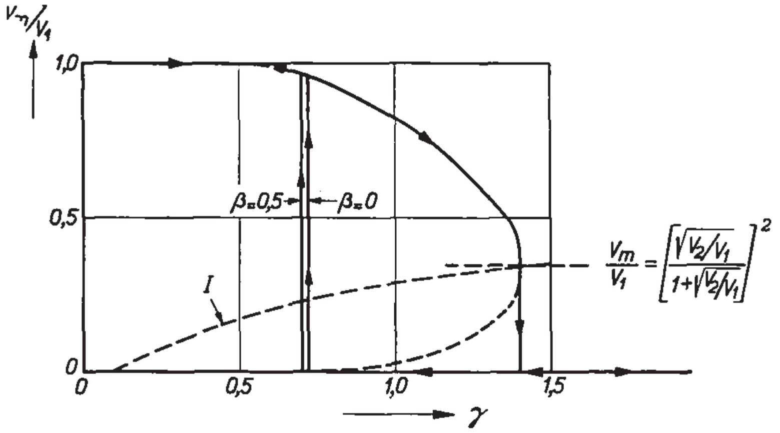

Fig. 5. V_m/V_1 as a function of γ for $V_2/V_1 = 2{,}0$. The horizontal scale has to be multiplied by 10.

Besides the potential V_m, the situation of the potential minimum between the grid and the anode is equally important. The distance x_1 of the minimum from the grid is shown in fig. 6 as a function of V_2/V_1 for $\gamma = 6$. If V_2/V_1 is greater than about 2,2 we have $x_1 = 0$. If V_2/V_1 is then decreased, x_1 increases steadily until V_2/V_1 is about 0,66. Here a virtual cathode with $V_m = 0$ must arise. In the whole region between V_2/V_1 about 1,63 and 0,66 (for $\beta = 0$) two situations of the potential minimum are possible, one with $V_m = 0$ (virtual cathode) and one with V_m finite. Transition from one state of affairs to the other may occur at any point in this region. The boundary curves I and II, which numerically coincide so nearly that no difference can be shown in fig. 6, determine the abrupt transition points of the x_1-curves (see figs. 1 and 1*a*). Figs. 7 and 8 are similar to fig. 6 but hold for $\gamma = 2{,}5$ and for $\gamma = 1$.

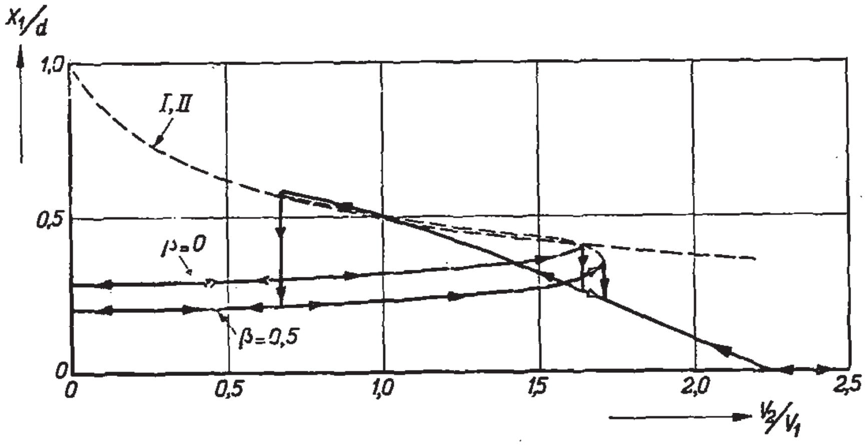

Fig. 6. Distance x_1 (cm) of the potential minimum from the grid plane as a function of V_2/V_1 for $\gamma = 6$.

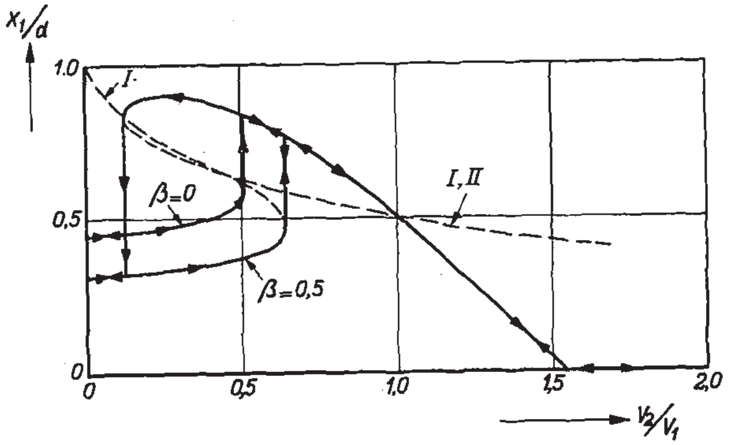

Fig. 7. As fig. 6 but for $\gamma = 2{,}5$.

IV. *Classification of the possible curve shapes.* In fig. 9 the possible curve shapes for i_2 as a function of V_2/V_1 for different values of γ are drawn schematically. It is remarkable, that in the case *d* no double-valued region for V_2/V_1 is present. This case, which corresponds to most of the practical values, encountered in actual vacuum tubes, is therefore representative for the curves to be measured in such valves if the electron velocity distribution is neglected.

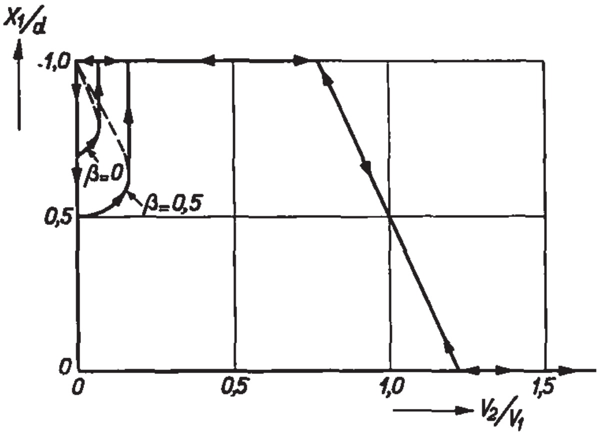

Fig. 8. As fig. 6 but for $\gamma = 1{,}0$.

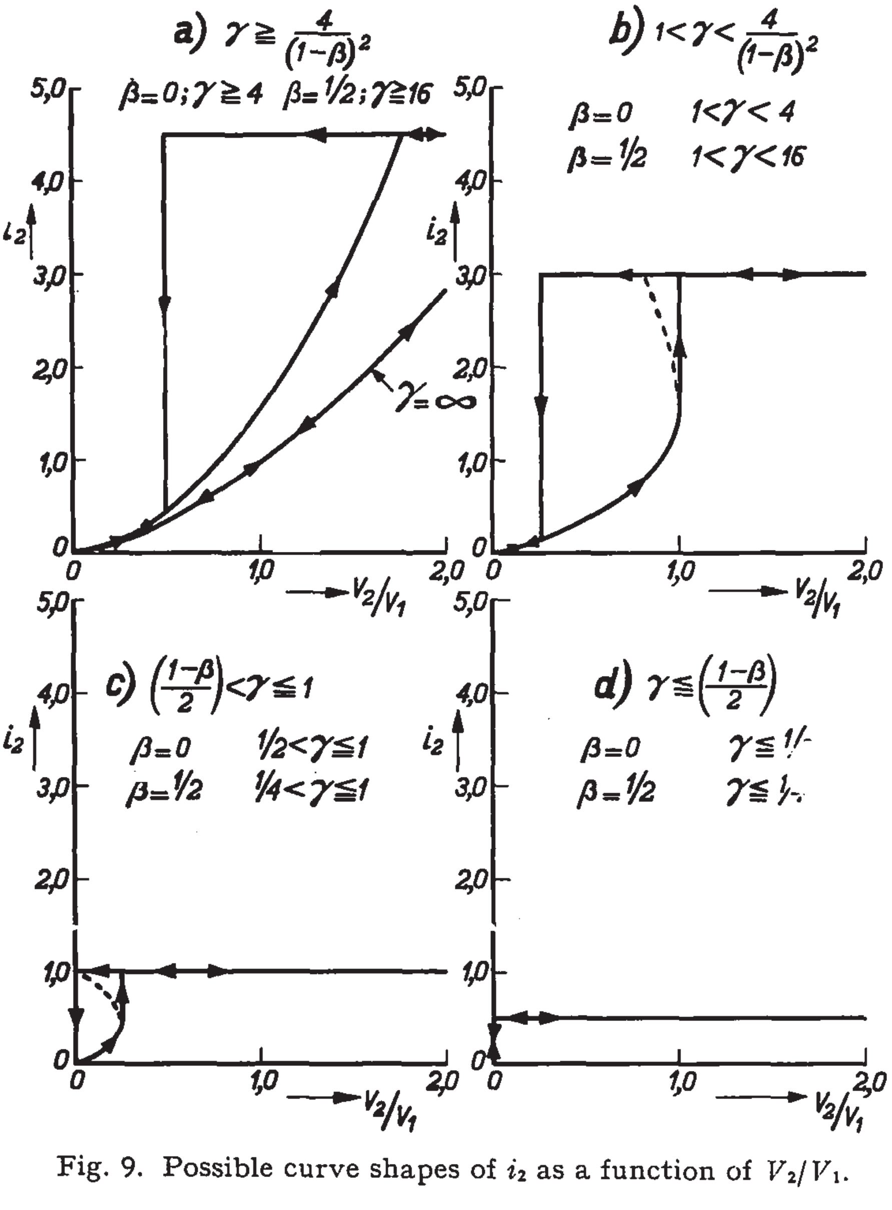

Fig. 9. Possible curve shapes of i_2 as a function of V_2/V_1.

Considering V_m/V_1 as a function of V_2/V_1 for different values of γ, 7 different curve shapes may be found (fig. 10). If γ is gradually increased, the successive curve shapes show the formation of a double valued region, which, for $\gamma = \infty$, is shifted to infinitely large values of V_2/V_1.

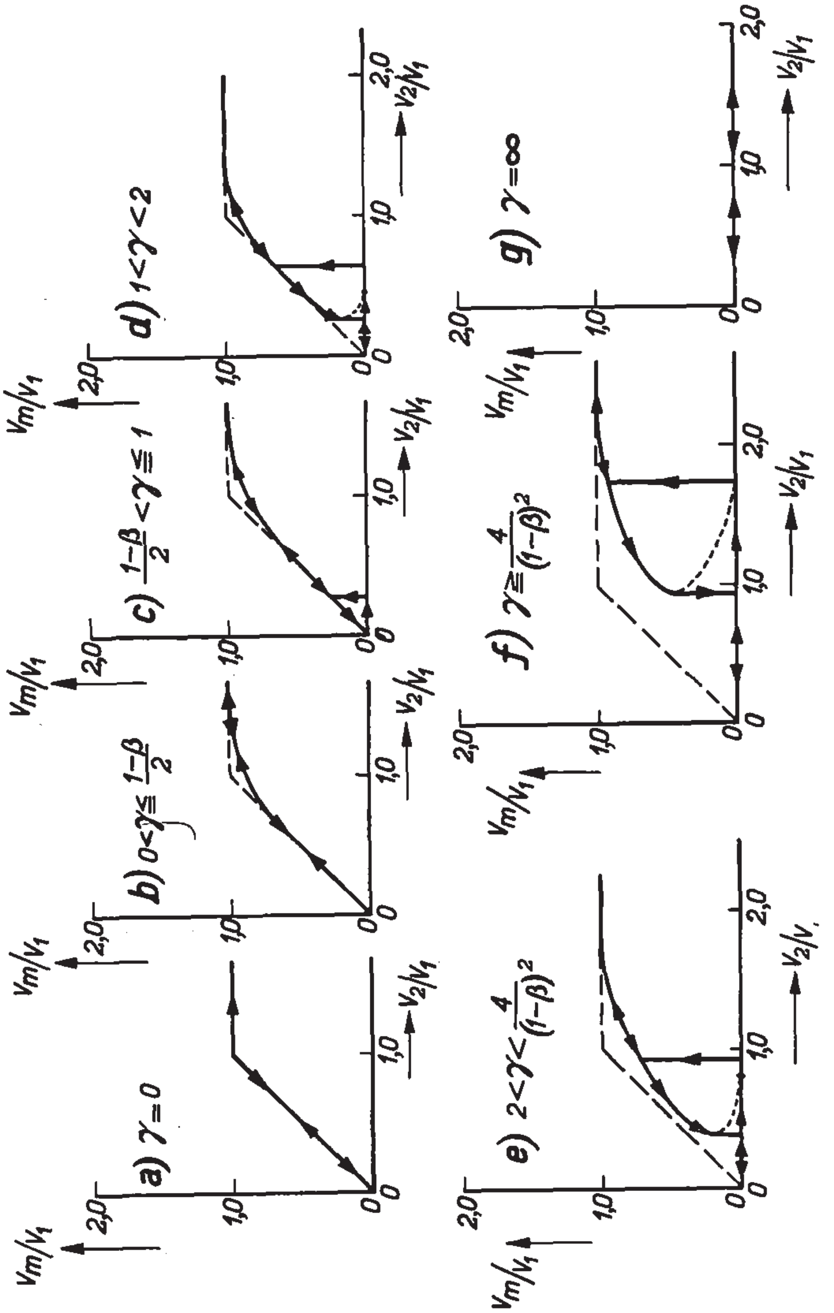

Fig. 10. Possible curve shapes of V_m/V_1 as a function of V_2/V_1.

In fig. 11 the distance x_1 of the potential minimum as a function of V_2/V_1 also gives rise to 7 different curve shapes. Here again, if

$0 < \gamma < (1 - \beta)/2$ no double valued region occurs, as is also seen in the case *b* of fig. 10.

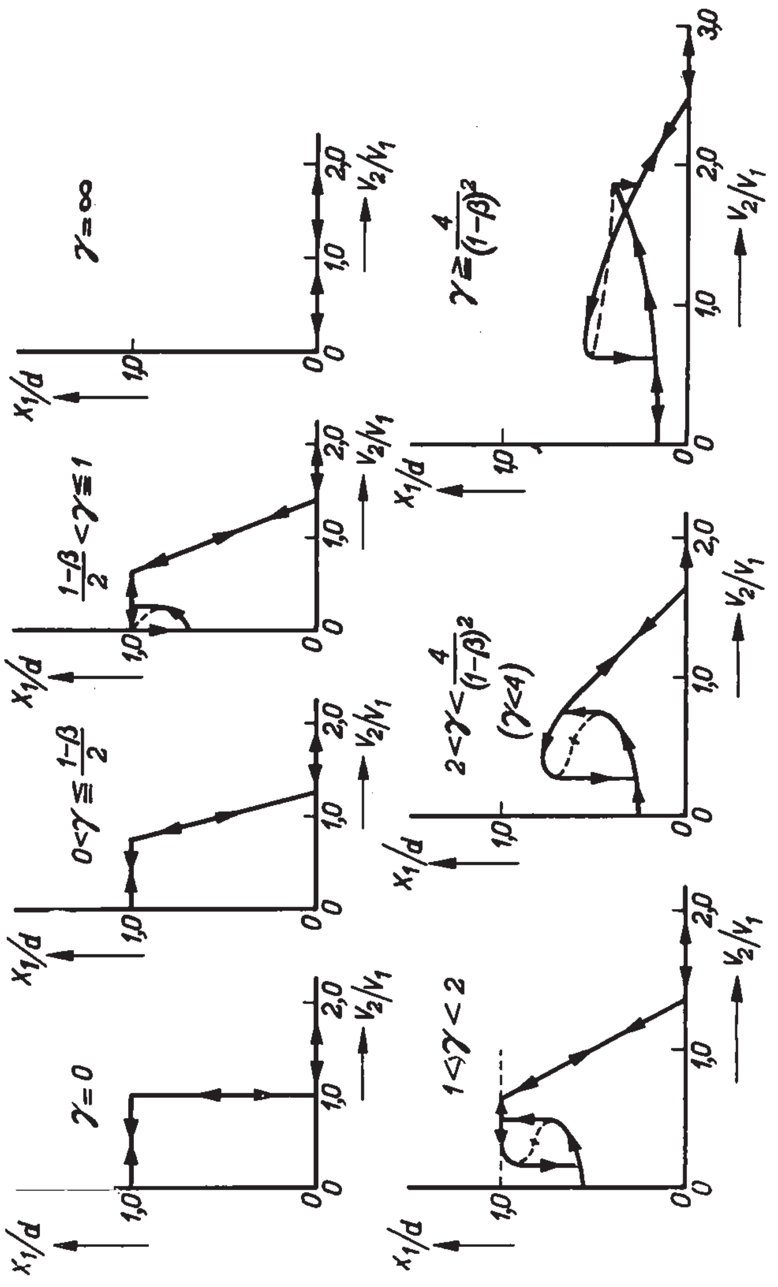

Fig. 11. Possible curve shapes of x_1 as a function of V_2/V_1.

By considering figs. 9, 10, 11, all essential features in the shapes of the characteristic curves become evident. We have also calculated and classified curves for i_2 as a function of γ, V_m as a function of γ and x_1 as a function of γ. These curves have, however, not been included in this discussion as they show no essentially new features besides those, contained in the figures considered here.

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